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Revisiting the Problem of Area Search Optimization for a Large Fleet of Heterogeneous Marine Unmanned Systems (MUS)

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Outline

- Introduction
- Mathematical Models
- Optimization Problem
- Results
- Discussion & Conclusion
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01. Introduction

Problem Definition

- NATO Nations developing diverse range of autonomous maritime platforms
- Area search/sterilization is a possible application
  - Dirty, **dangerous** & **dull**
- Need to develop methods
  - To quantify area search performance
  - To optimize fleet mix

Approach

- Reformulate the Anti-Submarine Warfare (ASW) area search as an optimization problem
- Utilize closed form search theory models
  - Enables rapid assessment
- Identify trends in scaled-up operations for MUS
- Assess the stability of results through stochastic variation of parameters
- Aim: optimize the fleet mix
  - Not a specific solution (*i.e.* X type Frigate for Y x Z km^2 of area)
  - But rather ***an extendable***, ***mathematically rigorous*** problem-solving framework

02. Mathematical Models

Assumptions

The following assumptions are valid for this analysis

- Search fleet already deployed and functioning in steady state
- Searchers have enough endurance to carry out the task
- Detection information is immediately available
- Target’s and searchers' speeds are constant
- Random movement
- Cookie cutter sensor with fixed detection range
- Target does not counter-detect and evade

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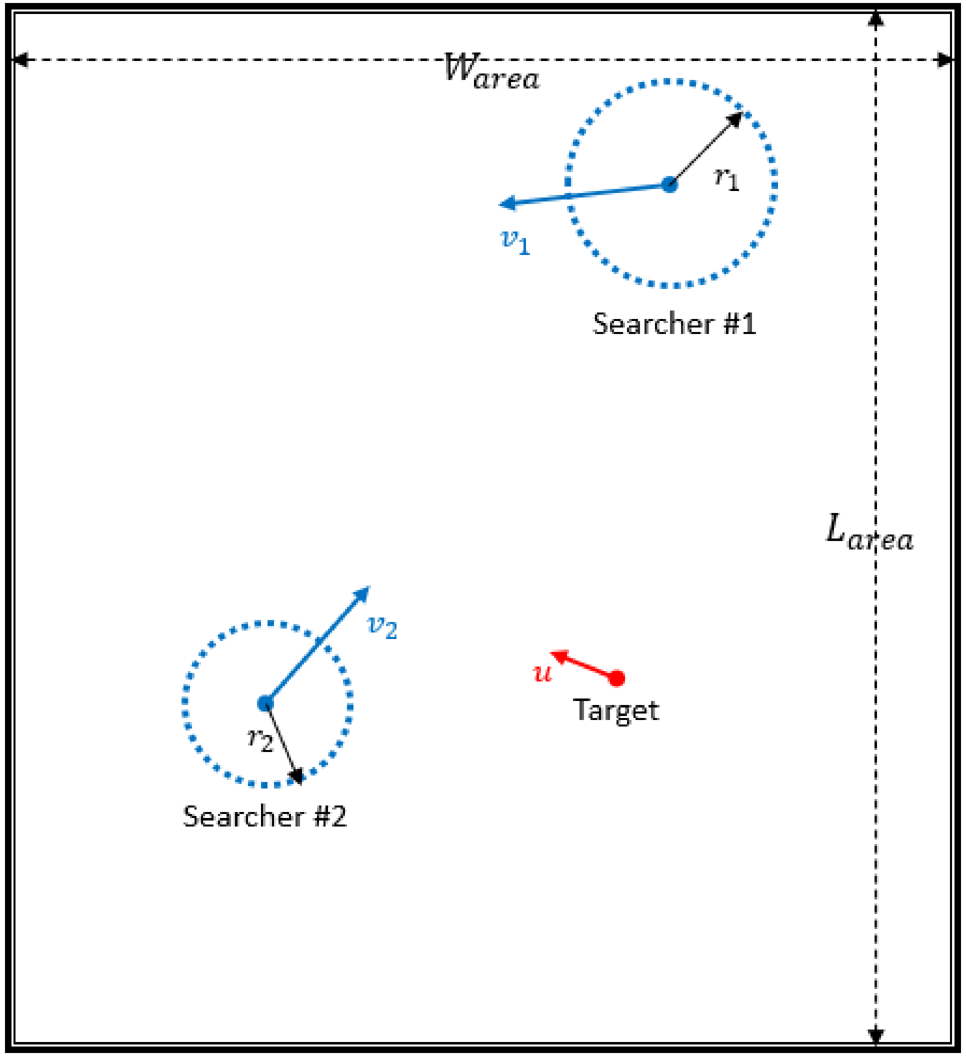
Models for Calculating Probability of Detection

- Cumulative probability of detection (CPD)
  - $\mathrm{CPD} = 1 - \prod_{i=1}^{N}(1 - p_i)$
- Type j assets with probability of detection p_j
  - $\mathrm{CPD} = 1 - \prod_{j=1}^{m}(1 - p_j)^{n_j}$
- Random search
  - $p_j(t) = 1 - e^{-t\frac{W_j v_j^{eq}}{A}}$
    - $v_j^{eq} = \frac{1}{2\pi}\int_0^{2\pi}\sqrt{u^2 + v_j^2 - 2uv_j\cos(\theta)}\,d\theta$
    - $W_j = 2r_j$



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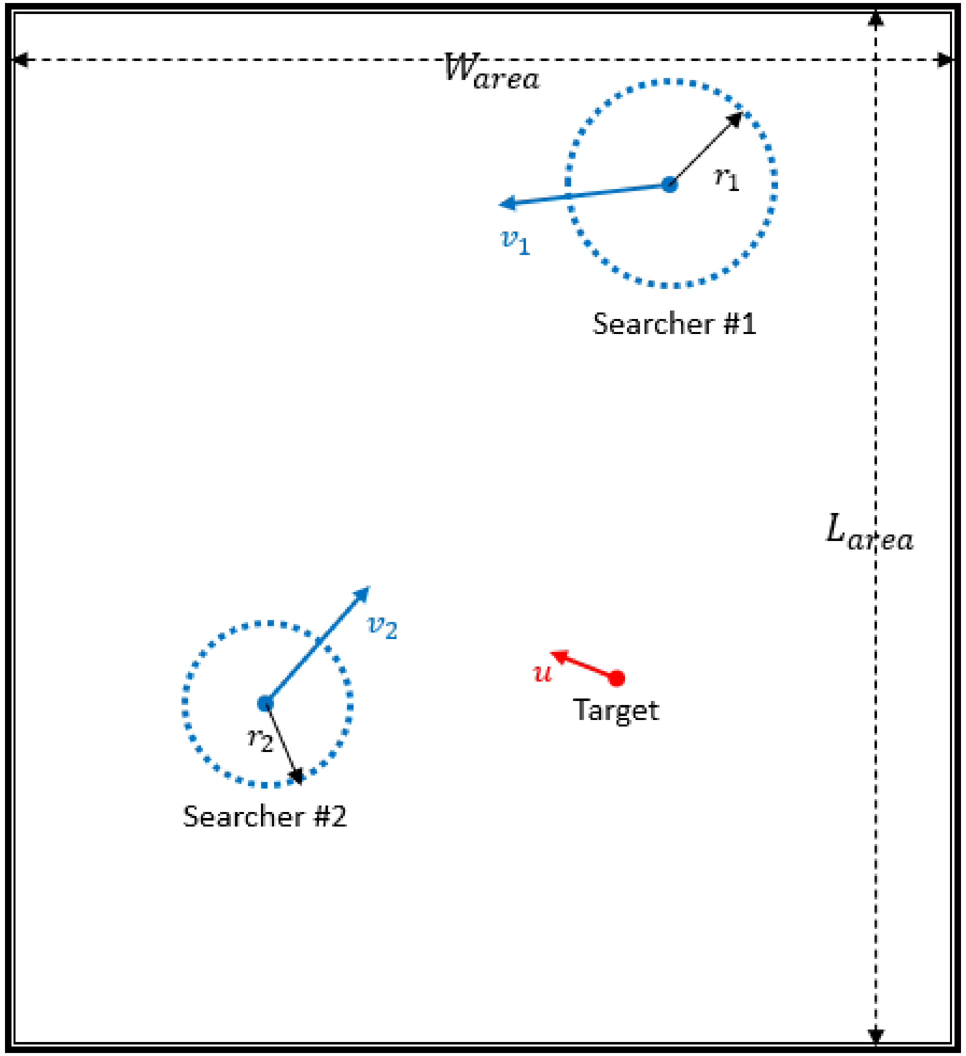
Models for Calculating Probability of Detection

- Re-arranging
  - $\ln(1-CPD) = -\frac{t}{A}\sum_{j=1}^{m} n_j W_j v_j^{eq}$
- Linear algebraic form
  - $\ln(1-CPD) = -\frac{t}{A}\begin{bmatrix} W_1 v_1^{eq} & \dots & W_m v_m^{eq} \end{bmatrix}\begin{bmatrix} n_1 \\ \vdots \\ n_m \end{bmatrix}$
- Hadamard product
  - $\ln(1-CPD) = -\frac{t}{A}\left(\begin{bmatrix} W_1 \\ \vdots \\ W_m \end{bmatrix} \circ \begin{bmatrix} v_1^{eq} \\ \vdots \\ v_m^{eq} \end{bmatrix}\right)^T \begin{bmatrix} n_1 \\ \vdots \\ n_m \end{bmatrix}$
- Direct expression of CPD
  - $\mathrm{CPD(t)} = 1 - \mathrm{e}^{-\frac{t}{A}(\overline{W}\circ\overline{v}^{eq})^T\overline{n}}$



03. Optimization Problem

Problem Synthesis

Pick a fleet mix defined by $\begin{bmatrix} n_1 & \cdots & n_m \end{bmatrix}^T = \bar{n}$

- The cost function is minimized $C_{cost}(\bar{n}) = \bar{c}^T \bar{n} = \begin{bmatrix} c_1 & \cdots & c_m \end{bmatrix} \begin{bmatrix} n_1 \\ \vdots \\ n_m \end{bmatrix}$
- Subject to constraints

$$-\frac{t_{search}}{A} (\bar{W} \circ \bar{v}^{eq})^T \bar{n} \leq \ln(1 - CPD^*)$$ →Desired CPD within t_{search}

$$\bar{0} \leq \bar{n} \leq \bar{n}^a$$ →Available assets

$$\bar{s}^T \bar{n} \leq 0$$ →Support capability

$$\bar{n} \in [0, \mathbf{Z}^+)$$ →Integer constraint

Constraints

- Area Search
  - Contributions from each of the type of assets shall add up or surpass the minimum required CPD^* within search time t_{search}

$$-\frac{t_{search}}{A}(\bar{W} \circ \bar{v}^{eq})^T \bar{n} \leq \ln(1 - CPD^*)$$

- Integer constraint
  - Can not have “half a Frigate”

Constraints

- Support capability
  - MUS have to be supported by “motherships”
    - Frigate and Support Vessel in our scenario
    - Support entails the capacity to enable mission, *i.e.* launch, recovery and replenishment
  - Assume support vessel can host
    - OR(*6* LUUVs, *8* MUUVs, *2* LUSVs, *10* SUUVs, *12* SGLDs, *16* UGLDs, *2* MUSVs)
    - Frigate has ½ capacity of the support vessel

$$[s_1 \quad \cdots \quad s_m]\begin{bmatrix} n_1 \\ \vdots \\ n_m \end{bmatrix} \leq 0$$

$\rightarrow s_j$: support coefficient

Constraints

- Defined by h_k^j
  - Depends on whether the asset is "host" or "guest" (*i.e.* support capable or not)
- If guest, maximum number of type *j* vehicles that can be held by the largest host of type *k*
- If host, capacity ratio of smaller type *j* host to the largest host of type *k*

$$s_j = \begin{cases} -\frac{1}{h_k^j}, & \text{if j is host} \\ 0, & \text{if j is neither guest or host} \\ \frac{1}{h_k^j}, & \text{if j is guest} \end{cases}$$

Constraints

- The fleet mix shall not have more vehicles than the ones available to the mission planner

$$\bar{0} \leq \bar{n} \leq \bar{n}^a$$

- Where

$$\begin{bmatrix} n_1^a & \cdots & n_m^a \end{bmatrix}^T = \bar{n}^a$$

$$\begin{bmatrix} n_1 & \cdots & n_m \end{bmatrix}^T = \bar{n}$$

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03. Results

MUS Data

- Data generic in nature
  - Not a rigorous costing study
- Main aim
  - Showcase the difference among assets

Vessel Type	Cruise Speed (kts)	Detection Range (km)	Operation Cost ($/hr)
Frigate	*9*	*15*	*14000*
Support Ship	*10*	*0*	*7100*
LUUV	4	8	4000
MUUV	3.5	6	1000
LUSV	10	10	800
SUUV	3	5	50
SGLD	1	3	7
UGLD	1	1	4
MUSV	9	8	700
SSBN	5	10	20000

Acronyms:

Large UUV (LUUV), Medium UUV (MUUV), Small UUV (SUUV), Large Unmanned Surface Vehicles (LUSV), Medium USV (MUSV), Surface Glider (SGLD), Underwater Glider (UGLD), Nuclear Submarine (SSBN)

Example Optimization Case

- Parameters
  - Square area with sides of *200* km
  - Target speed at *4* knots
  - *CPD** is *0.90* within *24* hours
- Following task force available →

Frigate	Support	LUUV	MUUV	LUSV	SUUV	SGLD	UGLD	MUSV	SSBN

$$\bar{n}^a = \begin{bmatrix} 3 & 3 & 6 & 12 & 6 & 24 & 24 & 45 & 6 & 0 \end{bmatrix}^T$$

- Optimized fleet mix
  - 21 SUUVs, 24 SGLDs. 1 UGLD, 2 Frigates & 3 support vessels
  - Validated with a brute force optimizer
    - Checks every possible combination there is
    - Albeit with x1000 processing time

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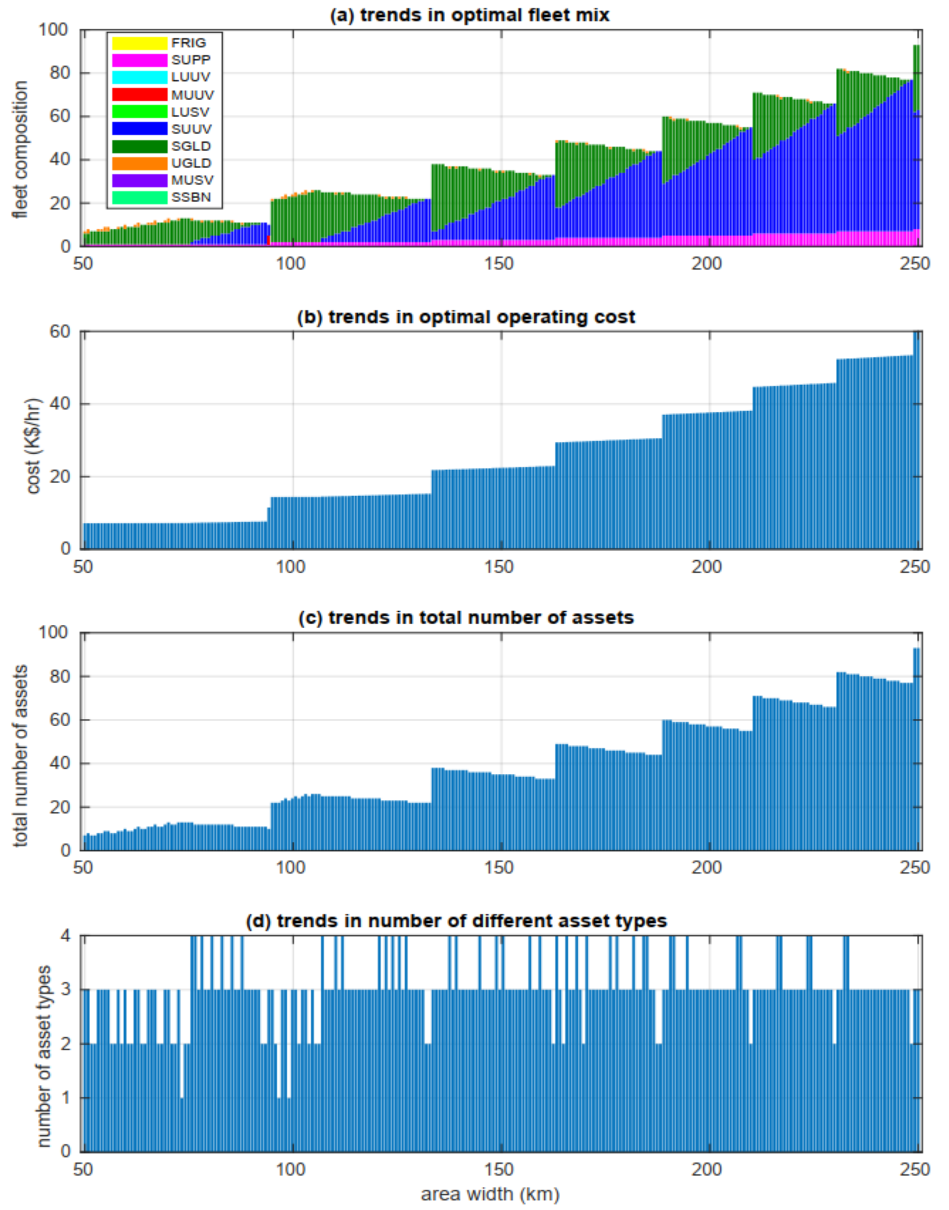
Trend Analysis – Increasing Search Area

- Optimized for cost
- Square search area
  - Side 50 km → 250 km
- Target speed at *4* knots
- *CPD** is *0.90* within *24* hours
- Unlimited number of assets



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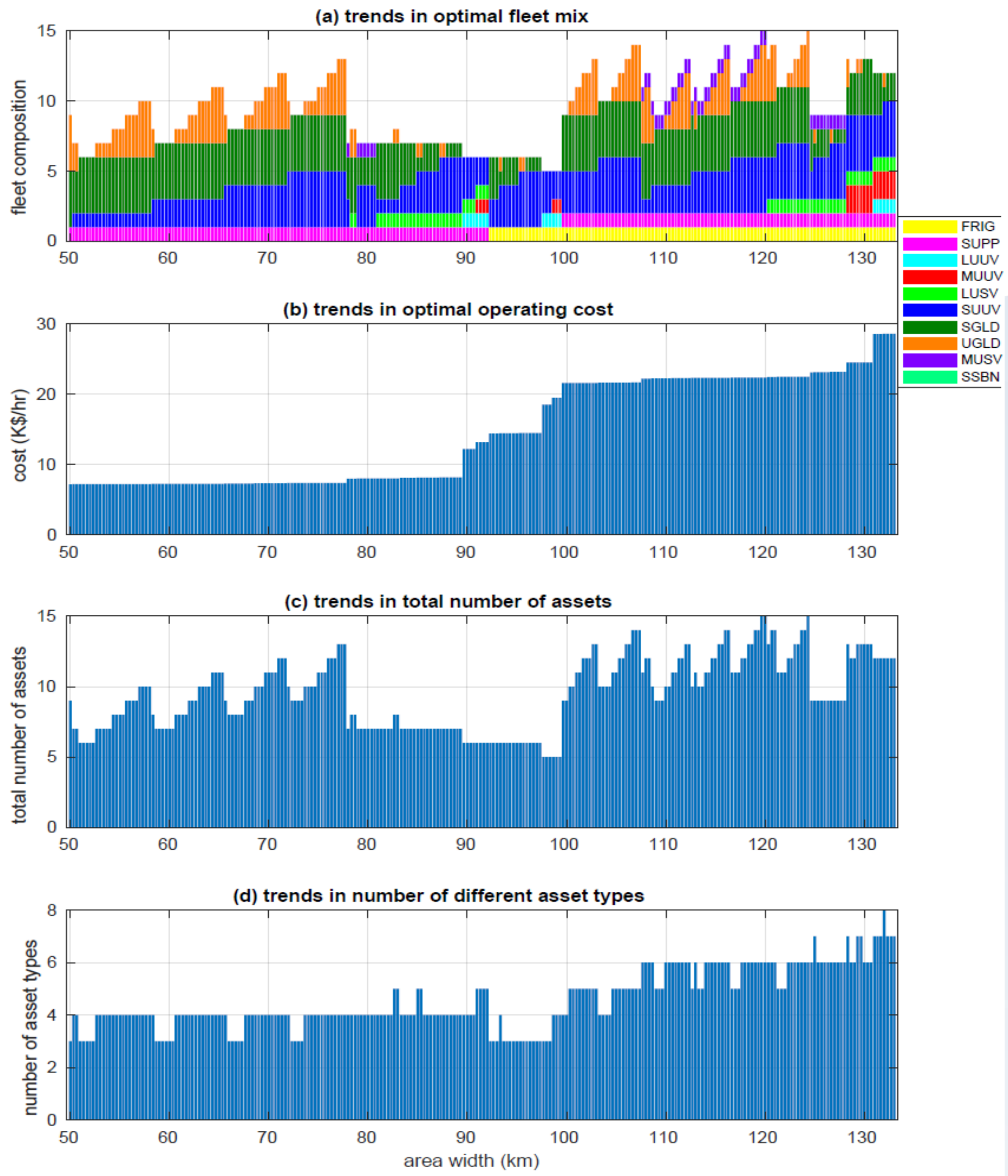
Trend Analysis – Limited Assets

Frigate	Support	LUUV	MUUV	LUSV	SUUV	SGLD	UGLD	MUSV	SSBN

$$\bar{n}^a = \begin{bmatrix} 1 & 1 & 1 & 2 & 1 & 4 & 4 & 4 & 2 & 0 \end{bmatrix}^T$$

- Same target speed & probability of detection
- Optimized for cost
- Area width 50 km → 133 km
  - Turns out 133^2 km^2 is the maximum possible for the given task force



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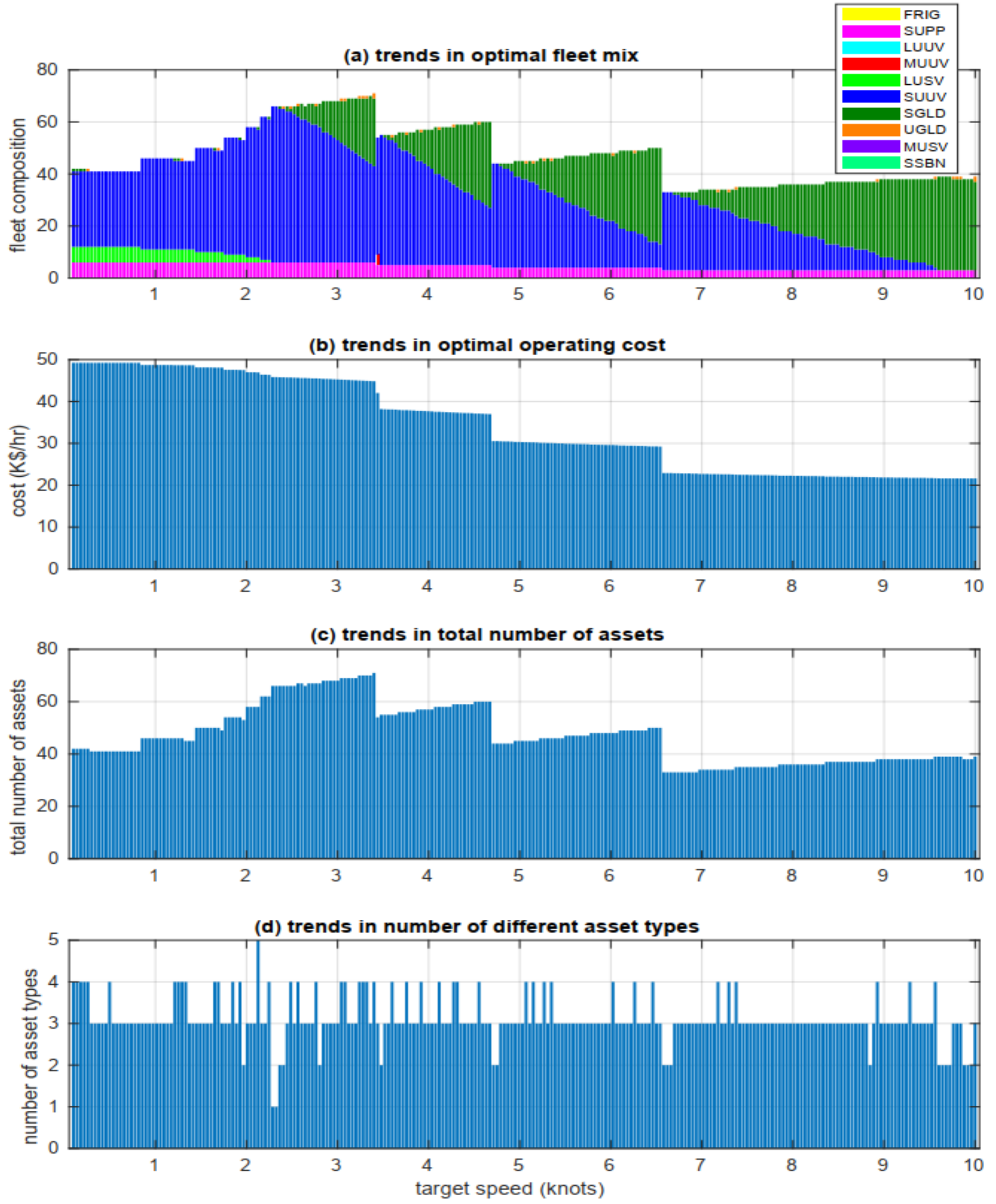
Trend Analysis – Target Speed

- Optimized for cost
- Square search area, 200 km side
- Target speed at 0 kts → 10 kts
- *CPD** is *0.90* within *24* hours
- Unlimited number of assets



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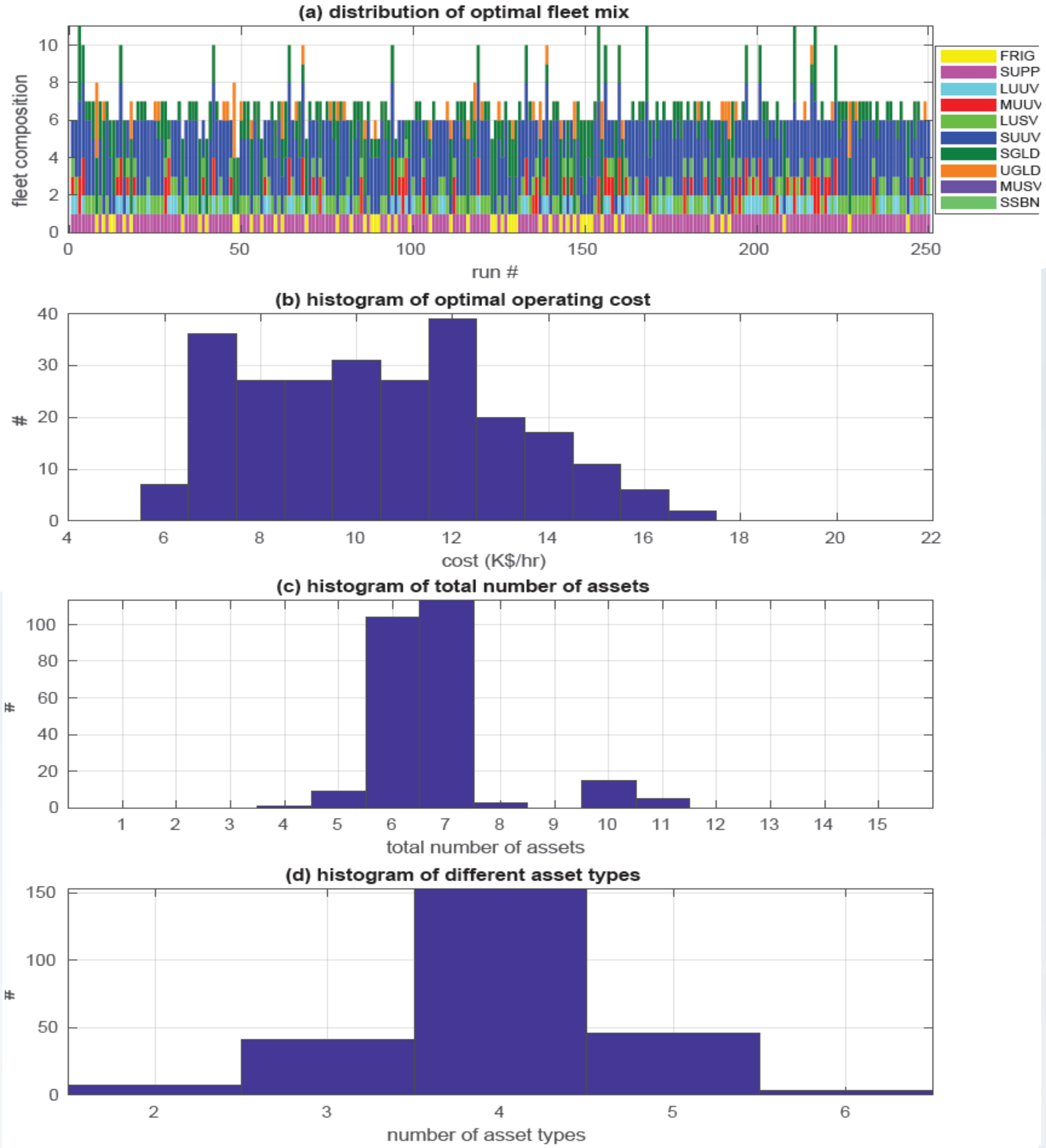
Sensitivity Analysis

- Parameters are stochastically distributed
  - Cost → Uniform Dist
    - Ranging from several fold to order of magnitude
  - Range & cruise speed → Normal Dist
    - Distributed over ±20%
- Same target speed & probability of detection as before
- Search area 90 km x 90 km, 250 iterations



04. Discussion & Conclusion

Discussion

- Formal mathematical definition of the area search optimization problem
  - Linear algebraic form $\hat{A}\bar{x} \leq \bar{b}$
  - Expandable with additional constraints
  - Can be reframed as a fleet-wise control problem

$$\dot{\bar{x}} = \hat{A}\bar{x} + \hat{B}\bar{u}$$

$$\bar{y} = \hat{C}\bar{x} + \hat{D}\bar{u}$$

- Integer linear programming formulation
  - Compliant with commercial-off-the-shelf (COTS) open source solvers with rapid calculation
  - An agnostic problem-solving framework
- Several limitations have to be addressed in the future
  - All events assumed to be independent
    - What if reactive target?
  - Perfect communication, the detection info is immediately relayed back to the task force commander
  - No sonar range prediction

Discussion

- Utilization of MUS can lead to more cost effective solutions in contrast to the traditional ones for area search
  - Step-like behavior in cost while scaling
  - More costly to scale-up in contrast to barrier
- Can be remedied by
  - Lowering the cost of logistics support platform
  - Enhancing the MUS logistics capacity of already deployed traditional platform
  - Designing the MUS platforms for coast or harbor self-launch and recovery
- Even though there are *10* different types of MUS, optimized solutions generally preferred *3* or *4* different types
  - Build more specific mission oriented MUS
  - Minimal loss to the performance while reducing complexity

Conclusion

- A rigorous mathematical framework for the optimization of unmanned platforms performing ASW missions
  - Rapid determination of optimum force mix
  - Solver-agnostic
  - Different open source tools can be utilized
- Enabled quick evaluation of numerous optimization problems & sensitivity analyses to assess uncertainty
  - Cost structure of area search optimized solutions are determined
- An individual MUS may cost orders of magnitude less **but** the ***support requirements may in turn dominate the cost***
  - Could be resolved through the design of
    - Individual MUS; *i.e.* designing more independent vehicles requiring less service
    - The systems of systems compromised of MUS; *i.e.* algorithms that optimize servicing, deployment etc.

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